Effect of vorticity on second- and third-order statistics of passive scalar gradients

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The influence of vorticity on second- and third-order moments of the spatial derivatives of a forced, passive scalar field has been studied in the framework of a simplified problem; the analysis is restricted to dominating rotation and molecular diffusion is represented by a linear model. The results reveal that, in the case of a passive scalar experiencing forcing in an isotropic medium, both vorticity and diffusion counteract anisotropy imposed on the scalar field. Anisotropy at the level of second-order moments appears to be destroyed essentially by the action of vorticity.

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I. INTRODUCTION

The question of the connection between large and small scales is essential to the field of turbulence. In this respect, determining to what extent small scales experience the influence of large ones as well as the way in which such a ''contamination'' takes place is an important task [1–3]. Especially, evaluation of small-scale anisotropy implied by anisotropic large-scale forcing of velocity or scalar fields has led to seriously question the principle of local isotropy [4–11]. As a matter of fact, there is now some evidence that large and small scales can be directly connected [1,11–13]. The scenario of such a coupling has been explained, in particular, in the case of a passive scalar field forced by a large-scale gradient; in this situation, experiments as well as numerical simulations indeed show that sharp scalar sheets (or ''cliffs''), aligned normally to forcing under influence of the large-scale flow, cause violation of the small-scale isotropy of the scalar field [3,6–8,12,13].

An outstanding result of previous studies is the persistence of anisotropy with increasing turbulence Reynolds number [11,12]. This is suggested by the behavior of increments and gradient skewness but is also true at the level of second-order moments of scalar derivatives [8,10]; the latter can, moreover, display a significant anisotropy in the presence of shear [14–16]. Notwithstanding the fact that recent studies [17] reveal that the skewness may not be a relevant indicator of a possible tendency toward isotropy, such features raise further questions, especially, about mechanisms that may oppose anisotropy.

Examination of the equations for the second-order moments of scalar derivatives [18] shows that isotropization can be ascribed only to molecular dissipation and/or stretching. Stretching acts on scalar gradients through strain and rotation. On the one hand, in the presence of a large-scale forcing, strain is likely to cause anisotropy of second- and third-order moments of scalar derivatives via compression along the direction of forcing. On the other hand, the significant influence of rotation on second-order statistics of scalar derivatives has been proved by numerical simulations in the case of sheared turbulence [19] and a recent experimental and modeling study [20] suggests that vorticity may also promote isotropization at this level. The particular action of vorticity thus deserves to be studied regarding its possible action in opposing the anisotropy imposed by strain.

Previous studies [21,22] have addressed the respective actions of strain and vorticity on scalar dissipation $D\langle g_\alpha g_\alpha\rangle$ (where D is the molecular diffusivity of the scalar, g_i the fluctuating gradient component in direction i, and $\langle\cdot\rangle$ represents the mean). The present one is focused on the effect of vorticity on second-order moments $\langle g_i g_j\rangle$ and third-order moments $\langle g_i^3\rangle$ in the case of a forced scalar field.

First, a qualitative analysis of vorticity effects is reported. Then, second- and third-order statistics of scalar gradient components are studied in the framework of a simplified situation that is assumed to mimic rotation-dominated regions in a flow. The dissipation process implied by molecular diffusion is accounted for by means of linear modeling.

II. QUALITATIVE ANALYSIS

The gradient $\boldsymbol{G}$ of a passive scalar embedded in a fluid flow experiences the effects of strain, rotation, and molecular diffusion [19,21–23]. The combined actions of these mechanisms are expressed by the equations for the components G_i of the scalar gradient

$$\frac{dG_i}{dt}=-S_{i\alpha}G_\alpha-\frac{1}{2}\omega_\alpha\varepsilon_{\alpha i\beta}G_\beta+D\frac{\partial^2 G_i}{\partial x_\alpha \partial x_\alpha}, \tag{1}$$

where S_{ij} represents the components of strain and ω_i the components of vorticity. The scalar diffusivity D is assumed to be constant; ε_{ijk} is the alternating symbol. The fluctuations of component G_i with respect to its mean value $\langle G_i\rangle$ will be written as g_i (hence, $G_i=\langle G_i\rangle+g_i$).

Second-order moments of the fluctuating components of $\boldsymbol{G}$ are defined as

$$\mathcal{G}_{ij}=\langle g_i g_j\rangle. \tag{2}$$

Tensor $\boldsymbol{\mathcal{G}}$ is an important quantity with regard to small-scale mixing. In particular, $D\mathcal{G}_{\alpha\alpha}$ is the mean dissipation rate of the energy of scalar fluctuations. Moreover, the invariants of the anisotropy tensor formed from $\boldsymbol{\mathcal{G}}$ reveal the scalar field structure at the level of dissipation [16].

Let us consider an isotropic turbulent medium in which a passive scalar field is forced by a uniform mean gradient $\boldsymbol{\Gamma}$ in such a way that $\langle G_2\rangle=\Gamma$ ($\Gamma>0$, say) and $\langle G_1\rangle=\langle G_3\rangle=0$ [24]. The general equations for the components of $\mathcal{G}_{ij}$ can be derived from Eq. (1) [18]. The effect of vorticity on the diagonal components of $\boldsymbol{\mathcal{G}}$ in the present situation is assessed by analyzing the corresponding equations in which strain, dissipation, and transport terms are dropped:

$$\frac{d\mathcal{G}_{11}}{dt}=-2\langle\omega_3 g_1\rangle\Gamma+\langle\omega_2 g_1 g_3\rangle-\langle\omega_3 g_1 g_2\rangle, \quad (3)$$

$$\frac{d\mathcal{G}_{22}}{dt}=\langle\omega_3 g_1 g_2\rangle-\langle\omega_1 g_2 g_3\rangle, \quad (4)$$

$$\frac{d\mathcal{G}_{33}}{dt}=2\langle\omega_1 g_3\rangle\Gamma+\langle\omega_1 g_2 g_3\rangle-\langle\omega_2 g_1 g_3\rangle. \quad (5)$$

Indices 1, 2, and 3 refer to components in a coordinate system defined by the orthogonal vector base $(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3)$ with $\mathbf{x}_3=\mathbf{x}_1\times\mathbf{x}_2$. For convenience, it will be assumed that the scalar is represented by heat (in passive conditions) and that $\mathbf{x}_2$ direction is referred to as ''up'' and $-\mathbf{x}_2$ as ''down.'' The case under study thus corresponds to an isotropic turbulent flow in which a transverse, uniform temperature gradient is applied. This situation is akin to a well-documented experiment [8,10,25].

The forcing terms [first terms on the right-hand sides of Eqs. (3) and (5), respectively] represent production of $\mathcal{G}_{11}$ and $\mathcal{G}_{33}$. It is indeed easy to show that $\langle\omega_3 g_1\rangle$ is negative whereas $\langle\omega_1 g_3\rangle$ is positive. As a matter of fact, positive rotation around $\mathbf{x}_3$ $(\omega_3>0)$ simultaneously brings warm fluid toward $\mathbf{x}_1<0$ and cold fluid toward $\mathbf{x}_1>0$ causing a negative fluctuation of the gradient along $\mathbf{x}_1$, that is, $g_1<0$. Inversely, $\omega_3<0$ implies $g_1>0$ and hence, $\langle\omega_3 g_1\rangle<0$. A similar reasoning on the effect of rotation around $\mathbf{x}_1$ shows that $\langle\omega_1 g_3\rangle>0$. Vorticity, therefore, promotes the variance of gradient components that are normal to the direction of forcing (g_1 and g_3, in the present case). A negative forcing $(\Gamma<0)$ would lead to the same result since, in that case, correlations between vorticity and scalar gradient components would assume opposite signs.

The second effect of vorticity consists in redistributing the energy of gradient fluctuations among the three directions, which is represented by the third-order correlation terms. The sum of the latter is indeed zero as can be checked from Eqs. (3)–(5). In the present situation, redistribution ''feeds'' $\mathcal{G}_{22}$ at the expense of $\mathcal{G}_{11}$ and $\mathcal{G}_{33}$; this mechanism is represented by the terms $\langle\omega_3 g_1 g_2\rangle$ and $\langle\omega_1 g_2 g_3\rangle$, respectively. The former is essentially positive while the latter is negative. As already mentioned, ω_3 and g_1 are negatively correlated. In addition, any rotation around $\mathbf{x}_3$ tends to bring warm fluid down and cold fluid up, resulting in a negative fluctuation g_2 of the gradient along $\mathbf{x}_2$; this implies $\langle\omega_3 g_1 g_2\rangle>0$. Similarly, $\langle\omega_1 g_2 g_3\rangle<0$ (any rotation around $\mathbf{x}_1$ also causes a negative fluctuation g_2). Correlation $\langle\omega_2 g_1 g_3\rangle$ is zero because rotation around $\mathbf{x}_2$ (the direction of forcing) does not cause any fluctuation of the scalar gradient components. Hence, there is no redistribution between $\mathcal{G}_{11}$ and $\mathcal{G}_{33}$.

To summarize, it is likely that anisotropic large-scale forcing of a scalar field in isotropic turbulence results, in rotation-dominated regions, in promoting the diagonal components of $\boldsymbol{\mathcal{G}}$ corresponding to fluctuating gradients normal to forcing ($\mathcal{G}_{11}$ and $\mathcal{G}_{33}$, here). Vorticity also limits this effect through redistribution (here, from $\mathcal{G}_{11}$ and $\mathcal{G}_{33}$ toward $\mathcal{G}_{22}$). Interestingly, previous studies [7,8,10] have shown that the statistics of scalar gradients on the total flow revealed, on the contrary, that $\mathcal{G}_{22}>\mathcal{G}_{11}\simeq\mathcal{G}_{33}$. Since the latter result is most probably explained by the effect, on the scalar field, of compression events associated with cliff structures normal to the direction of forcing [8,18], it can be inferred that the statistics of scalar gradients is dominated by strain in this kind of experiment.

The influence of vorticity on the skewnesses of gradient components, that is, $S_{G_i}=\langle g_i^3\rangle$, can also be discussed from simplified equations. The latter are derived using Eq. (1),

$$\frac{dS_{G_1}}{dt}=-\frac{3}{2}\langle\omega_3 g_1^2\rangle\Gamma+\frac{3}{2}\langle\omega_2 g_1^2 g_3\rangle-\frac{3}{2}\langle\omega_3 g_1^2 g_2\rangle, \quad (6)$$

$$\frac{dS_{G_2}}{dt}=\frac{3}{2}(\langle\omega_3 g_1\rangle-\langle\omega_1 g_3\rangle)\Gamma^2+\frac{3}{2}(\langle\omega_3 g_1 g_2^2\rangle -\langle\omega_1 g_2^2 g_3\rangle), \quad (7)$$

$$\frac{dS_{G_3}}{dt}=\frac{3}{2}\langle\omega_1 g_3^2\rangle\Gamma-\frac{3}{2}\langle\omega_2 g_1 g_3^2\rangle+\frac{3}{2}\langle\omega_1 g_2 g_3^2\rangle. \quad (8)$$

In Eq. (6), $\langle\omega_3 g_1^2\rangle=0$ (because of ω_3 symmetry) and $\langle\omega_2 g_1^2 g_3\rangle=0$ (which results from the fact that ω_2 and the fluctuating components g_i are uncorrelated). Correlation $\langle\omega_3 g_1^2 g_2\rangle$ is also equal to zero. Hence, vorticity alone does not imply any skewness of G_1, which is expected because ω_3 makes fluctuations g_1, symmetrically, either positive or negative. Similarly, there is no production of S_{G_3}.

Examination of the terms of Eq. (7), following the same kind of reasoning as previously, reveals that $\langle\omega_3 g_1 g_2^2\rangle<0$ and $\langle\omega_1 g_2^2 g_3\rangle>0$; ω_3 and g_1 are negatively correlated whereas ω_1 and g_3 are positively correlated as already explained. Vorticity thereby produces a negative skewness S_{G_2} when $\Gamma>0$ and the opposite result would be found in the case of a negative forcing Γ. Vorticity thus implies a skewness of G_2, the sign of which is opposite to the sign of forcing. Now, it is established that in isotropic turbulence, $\mathrm{sgn}(S_{G_2})=\mathrm{sgn}(\Gamma)$ [5–8,25] as a result of the existence of cliff structures moving along the direction of forcing. This, again, lends support to the idea that scalar gradient statistics is determined by strain in this situation. In passing, it is worth noticing that the statistics of scalar gradients, at least at the level of second-order moments, is governed by vorticity rather than by strain when shear is superimposed on the forcing of the scalar field [19].

III. MODEL PROBLEM

A. Instantaneous scalar gradient

The role of vorticity in the statistics of scalar gradients is now investigated analytically in the framework of a simplified problem. As in the preceding section, the situation of a scalar field forced by a gradient $\mathbf{\Gamma}$ (parallel to $\mathbf{x}_2$) is considered. Equation (1) is written for G_1, G_2, and G_3 neglecting the influence of strain. In a way similar to the approach used for modeling the effect of viscosity in the velocity gradient equation [26], molecular diffusion is represented by a relaxation mechanism acting with the diffusion frequency f_D.

The following system, derived from Eq. (1) together with the above hypotheses, is subsequently assumed to mimic the evolution of the scalar gradient components when rotation dominates:

$$\frac{dG_1}{dt^*}=\frac{1}{2}(\omega_2^*G_3-\omega_3^*G_2)-f_D^*G_1, \tag{9}$$

$$\frac{dG_2}{dt^*}=\frac{1}{2}(\omega_3^*G_1-\omega_1^*G_3)-f_D^*(G_2-\Gamma), \tag{10}$$

$$\frac{dG_3}{dt^*}=\frac{1}{2}(\omega_1^*G_2-\omega_2^*G_1)-f_D^*G_3. \tag{11}$$

Vorticity decay due to viscosity is neglected and the diffusion frequency f_D is assumed not to depend on time. The following normalized quantities are defined: $t^*=\omega t$, $f_D^*=f_D/\omega$, $\omega_i^*=\omega_i/\omega$, with $\omega=(\omega_\alpha\omega_\alpha)^{1/2}$.

With $G_1(0)=G_3(0)=0$ and $G_2(0)=\gamma$ as initial conditions, the solution of Eqs. (9)–(11) is

$$G_i=[A_i\cos(t^*/2)+B_i\sin(t^*/2)+\omega_i^*\omega_2^*(\gamma-\Gamma)]\times\exp(-f_D^*t^*)+C_i, \tag{12}$$

where

$$A_1=-\omega_1^*\omega_2^*\gamma+\frac{2f_D^*}{1+4f_D^{*2}}(2f_D^*\omega_1^*\omega_2^*+\omega_3^*)\Gamma;$$

$$B_1=-\omega_3^*\gamma+\frac{2f_D^*}{1+4f_D^{*2}}(2f_D^*\omega_3^*\gamma-\omega_1^*\omega_2^*\Gamma),$$

$$A_2=\left(\gamma-\frac{4f_D^{*2}}{1+4f_D^{*2}}\Gamma\right)(1-\omega_2^{*2});$$

$$B_2=\frac{2f_D^*}{1+4f_D^{*2}}(1-\omega_2^{*2})\Gamma,$$

$$A_3=-\omega_2^*\omega_3^*\gamma+\frac{2f_D^*}{1+4f_D^{*2}}(2f_D^*\omega_2^*\omega_3^*-\omega_1^*)\Gamma;$$

$$B_3=\omega_1^*\gamma-\frac{2f_D^*}{1+4f_D^{*2}}(2f_D^*\omega_1^*\gamma+\omega_2^*\omega_3^*\Gamma),$$

$$C_1=\frac{\omega_1^*\omega_2^*-2\omega_3^*f_D^*}{1+4f_D^{*2}}\Gamma;\quad C_2=\frac{\omega_2^{*2}+4f_D^{*2}}{1+4f_D^{*2}}\Gamma;$$

$$C_3=\frac{\omega_2^*\omega_3^*+2\omega_1^*f_D^*}{1+4f_D^{*2}}\Gamma.$$

It is to be stressed that the above solution, derived from a system in which the effect of strain is neglected, retains some physical validity with regard to the effect of pure rotation only if it is restricted to time smaller than the time scale of strain buildup. Since rotation-generated strain develops on a time scale of order ω^{-1}, this condition implies $t<\omega^{-1}$. In the following, the analysis will thereby be limited to $t^*<1$. When considering a turbulent medium, this restriction justifies neglecting vorticity destruction by viscosity effect since, at the smallest scales, $\omega_\ell^{-1}\sim\ell^2/\nu$ whereas at larger scales, $\omega_\ell^{-1}<\ell^2/\nu$ (where ω_ℓ is the vorticity at scale ℓ and ν the kinematic viscosity).

B. Second-order statistics of scalar gradient components

It is now assumed that vorticity has a random, isotropic orientation in the coordinate system $(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3)$ (see Appendix A). In other words, one considers a large number N ($N\to\infty$) of points at which a randomly oriented rotation is applied. In addition, it is stated that the diffusion frequency f_D^* is the same at each point. The statistics of the scalar gradient on this set of points is akin to an ensemble statistics conditioned on rotation. In this regard, it is to be underlined that the statistical properties derived in the following sections are thereby different from those relating to the total flow; in particular, the analytical results presented hereafter do not conflict with the discussion of Sec. II, which is devoted to the overall statistics. Note that symbol $\langle\cdot\rangle$ is, however, still used to represent the mean in the following.

From Eq. (12), it follows that

$$\langle G_1\rangle=0,\quad\langle G_3\rangle=0, \tag{13}$$

$$\langle G_2\rangle=\Gamma-\frac{(1-\langle\omega_2^{*2}\rangle)\Gamma}{1+4f_D^{*2}}\{1-[\cos(t^*/2)+2f_D^*\sin(t^*/2)]\exp(-f_D^*t^*)\}. \tag{14}$$

The above derivation of the mean gradient components requires the values of the vorticity moments (a number of them are zero—see Appendix A). It is also stated that $\langle\gamma\rangle=\Gamma$, which implies that, initially, the mean value of G_2 is Γ. Note that, from Eq. (14), $\langle G_2\rangle<\Gamma$. The statistics of G_3 is similar to the one of G_1 and is not reported in the following.

The second-order moments ($\langle g_1^2\rangle\equiv\mathcal{G}_{11}$, $\langle g_2^2\rangle\equiv\mathcal{G}_{22}$) are derived from Eqs. (12)–(14) and their exact expressions are given in Appendix B.

The case of vanishing rotation ($\omega\to0$) leads to obvious results since then

$$G_1=0\quad\text{and}\quad\mathcal{G}_{11}=0,$$

$$G_2=\Gamma+(\gamma-\Gamma)\exp(-f_Dt),$$

$$\mathcal{G}_{22}=\Gamma^2(\mathcal{I}_\gamma-1)\exp(-2f_Dt),$$

with $\mathcal{I}_\gamma=\langle\gamma^2\rangle/\Gamma^2$ (thus, $\mathcal{I}_\gamma\geqslant1$). If $\mathcal{G}_{22}$ is initially nonzero ($\mathcal{I}_\gamma\neq1$) then, the ratio $\mathcal{G}_{22}/\mathcal{G}_{11}$ remains infinite. In a more general case where $G_1(0)=\gamma_1$ and $G_2(0)=\gamma_2$ with $\langle\gamma_1\rangle=\Gamma_1$ and $\langle\gamma_2\rangle=\Gamma_2$, it is straightforward to show that

$\mathcal{G}_{22}/\mathcal{G}_{11}$ is constant. In the framework of the present model, diffusion alone, therefore, does not cause destruction of the initial anisotropy of second-order moments.

In the following, the regimes of vanishing and moderate diffusion determined, respectively, by $f_D^*\to 0$ and $O(f_D^*)=O(1)$, are considered, restricting the analysis to the small-time limit, $t^*\ll 1$.

1. $t^*\ll 1$ and $f_D^*\to 0$

From Eqs. (B1) and (B2), the following expressions are derived:

$$\mathcal{G}_{11}=\Gamma^2\mathcal{I}_\gamma\langle\omega_3^{*2}\rangle\frac{t^{*2}}{4}+O(t^{*4}), \tag{15}$$

$$\mathcal{G}_{22}=\Gamma^2(\mathcal{I}_\gamma-1)\left[1-(1-\langle\omega_2^{*2}\rangle)\frac{t^{*2}}{4}+\left(\frac{1}{3}-\frac{7}{12}\langle\omega_2^{*2}\rangle+\frac{1}{4}\langle\omega_2^{*4}\rangle\right)\frac{t^{*4}}{16}\right]+\Gamma^2(\langle\omega_2^{*4}\rangle-\langle\omega_2^{*2}\rangle^2)\frac{t^{*4}}{64}+O(t^{*6}). \tag{16}$$

Vorticity implies production of $\mathcal{G}_{11}$. The initial variance of G_2 is destroyed by vorticity that is represented by the $O(t^{*2})$ term in Eq. (16). Vorticity, however, also promotes $\mathcal{G}_{22}$ as revealed by both $O(t^{*4})$ terms.

The case $\mathcal{I}_\gamma=1$ is a special one that corresponds to zero initial variance of G_2. The variances of G_1 and G_2 thus become simplified,

$$\mathcal{G}_{11}=\Gamma^2\langle\omega_3^{*2}\rangle\frac{t^{*2}}{4}+O(t^{*4}), \tag{17}$$

$$\mathcal{G}_{22}=\Gamma^2(\langle\omega_2^{*4}\rangle-\langle\omega_2^{*2}\rangle^2)\frac{t^{*4}}{64}+O(t^{*6}). \tag{18}$$

The ratio $\mathcal{G}_{11}/\mathcal{G}_{22}$ is a measure of the diagonal anisotropy of tensor $\boldsymbol{\mathcal{G}}$,

$$\frac{\mathcal{G}_{11}}{\mathcal{G}_{22}}=16\frac{\langle\omega_3^{*2}\rangle}{\langle\omega_2^{*4}\rangle-\langle\omega_2^{*2}\rangle^2}t^{*-2}+O(1).$$

Replacing the moments of vorticity by their numerical values (see Appendix A),

$$\frac{\mathcal{G}_{11}}{\mathcal{G}_{22}}=60t^{*-2}+O(1). \tag{19}$$

Initially, the ratio $\mathcal{G}_{11}/\mathcal{G}_{22}$ tends to infinity. At small times $\mathcal{G}_{11}\gg\mathcal{G}_{22}$ but the ratio decreases rapidly as t^{*-2} under the action of vorticity. This is explained by the fact that $\mathcal{G}_{11}$ is produced via the interaction between vorticity and forcing and that $\mathcal{G}_{22}$ is fed by redistribution from $\mathcal{G}_{11}$ (and $\mathcal{G}_{33}$).

In the general case where $\mathcal{I}_\gamma\neq 1$,

$$\mathcal{G}_{11}=\Gamma^2\mathcal{I}_\gamma\langle\omega_3^{*2}\rangle\frac{t^{*2}}{4}+O(t^{*4}), \tag{20}$$

$$\mathcal{G}_{22}=\Gamma^2(\mathcal{I}_\gamma-1)\left[1-(1-\langle\omega_2^{*2}\rangle)\frac{t^{*2}}{4}\right]+O(t^{*4}), \tag{21}$$

and the evolution of anisotropy of second-order moments is given by

$$\frac{\mathcal{G}_{22}}{\mathcal{G}_{11}}=\frac{4(\mathcal{I}_\gamma-1)}{\mathcal{I}_\gamma\langle\omega_3^{*2}\rangle}t^{*-2}+O(1),$$

that is,

$$\frac{\mathcal{G}_{22}}{\mathcal{G}_{11}}=\frac{12(\mathcal{I}_\gamma-1)}{\mathcal{I}_\gamma}t^{*-2}+O(1). \tag{22}$$

In this case too, vorticity makes anisotropy decay as t^{*-2}.

2. $t^*\ll 1$ and $O(f_D^*)=O(1)$

In this regime,

$$\mathcal{G}_{11}=\Gamma^2\langle\omega_3^{*2}\rangle\left\{1+\frac{\mathcal{I}_\gamma-1}{(1+4f_D^{*2})^2}-f_D^*\left[1+2\frac{\mathcal{I}_\gamma-1}{(1+4f_D^{*2})^2}\right]t^*\right\}\frac{t^{*2}}{4}+O(t^{*4}), \tag{23}$$

$$\mathcal{G}_{22}=\Gamma^2(\mathcal{I}_\gamma-1)\left\{1-2f_D^*t^*+\left[2f_D^{*2}-\frac{1}{4}(1-\langle\omega_2^{*2}\rangle)\right]t^{*2}-2f_D^*\left[\frac{2}{3}f_D^{*2}-\frac{1}{4}(1-\langle\omega_2^{*2}\rangle)\right]t^{*3}+O(t^{*4})\right\}+\Gamma^2(\langle\omega_2^{*4}\rangle-\langle\omega_2^{*2}\rangle^2)\frac{t^{*4}}{64}+O(t^{*5}). \tag{24}$$

In Eq. (24), the first $O(t^{*4})$ term is not given explicitly because it makes the expression more complicated and is of no use for subsequent calculations.

The variance $\mathcal{G}_{11}$ is essentially produced by vorticity, at order $O(t^{*2})$; simultaneously, diffusion opposes the increase of $\mathcal{G}_{11}$ but at order $O(t^{*3})$. Diffusion causes the decay of the initial value of $\mathcal{G}_{22}$ via a $O(t^*)$ term. The last $O(t^{*4})$ term in Eq. (24) expresses promotion of $\mathcal{G}_{22}$ by vorticity.

If $\mathcal{I}_\gamma=1$ then

$$\mathcal{G}_{11}=\Gamma^2\langle\omega_3^{*2}\rangle(1-f_D^*t^*)\frac{t^{*2}}{4}+O(t^{*4}), \tag{25}$$

$$\mathcal{G}_{22}=\Gamma^2(\langle\omega_2^{*4}\rangle-\langle\omega_2^{*2}\rangle^2)\frac{t^{*4}}{64}+O(t^{*5}), \tag{26}$$

and

$$\frac{\mathcal{G}_{11}}{\mathcal{G}_{22}}=16\frac{\langle\omega_3^{*2}\rangle}{\langle\omega_2^{*4}\rangle-\langle\omega_2^{*2}\rangle^2}t^{*-2}+O(t^{*-1}),$$

which can be rewritten as

$$\frac{\mathcal{G}_{11}}{\mathcal{G}_{22}}=60t^{*-2}+O(t^{*-1}). \tag{27}$$

If $\mathcal{I}_\gamma\neq 1$, $\mathcal{G}_{11}$ is given by Eq. (23) and $\mathcal{G}_{22}$ by

$$\mathcal{G}_{22}=\Gamma^2(\mathcal{I}_\gamma-1)(1-2f_D^*t^*)+O(t^{*2}). \tag{28}$$

Then, from Eqs. (23) and (28),

$$\frac{\mathcal{G}_{22}}{\mathcal{G}_{11}}=\frac{4(\mathcal{I}_\gamma-1)t^{*-2}}{\langle\omega_3^{*2}\rangle\left[1+\dfrac{\mathcal{I}_\gamma-1}{(1+4f_D^{*2})^2}\right]}\left[1-\frac{f_D^*t^*}{1+\dfrac{\mathcal{I}_\gamma-1}{(1+4f_D^{*2})^2}}\right]+O(1). \tag{29}$$

At $t^*=0$, $\mathcal{G}_{11}=0$ and $\mathcal{G}_{22}=\Gamma^2(\mathcal{I}_\gamma-1)$ as a consequence of the initial conditions that have been chosen. Afterwards, $\mathcal{G}_{11}$ increases as a result of production by vorticity and $\mathcal{G}_{22}$ decreases essentially under action of diffusion [Eq. (28)]; the ratio $\mathcal{G}_{22}/\mathcal{G}_{11}$ decreases as t^{*-2}, at the leading order [Eq. (29)]. Diffusion also causes destruction of anisotropy but at order $O(t^{*-1})$. It is, however, clear from Eq. (29) that diffusion alone would not ensure the destruction of the initial anisotropy and that this process requires the existence of vorticity.

C. Third-order statistics of scalar gradient components

The third-order moments are derived from Eqs. (12)–(14). It is found that $\mathcal{S}_{G_1}=\mathcal{S}_{G_3}=0$ which was expected. The exact expression for $\mathcal{S}_{G_2}$ is given in Appendix B.

In the special case where $\mathcal{I}_\gamma=1$, Eq. (B3) is reduced to

$$\mathcal{S}_{G_2}=\frac{\Gamma^3}{(1+4f_D^{*2})^3}[\langle\omega_2^{*2}\rangle(3\langle\omega_2^{*4}\rangle-2\langle\omega_2^{*2}\rangle^2)-\langle\omega_2^{*6}\rangle]\times[\{\cos(t^*/2)+2f_D^*\sin(t^*/2)\}\exp(-f_D^*t^*)-1]^3, \tag{30}$$

that is, replacing the moments of vorticity by their numerical values,

$$\mathcal{S}_{G_2}=\frac{16\Gamma^3}{945(1+4f_D^{*2})^3}[1-\{\cos(t^*/2)+2f_D^*\sin(t^*/2)\}\times\exp(-f_D^*t^*)]^3. \tag{31}$$

Equation (31) shows that the sign of $\mathcal{S}_{G_2}$ is given by the sign of the forcing term, Γ (the cubic time-dependent term is positive). This is not inconsistent with the discussion of Sec. II since, here, $\langle G_2\rangle$ is an ensemble mean value that is not constant but, in the domain of interest ($t^*<1$), decreases under the action of vorticity as can be checked from Eq. (14).

Using Eq. (B2) (with $\mathcal{I}_\gamma=1$) and Eq. (31), the normalized skewness of G_2 is derived to be

$$\mathcal{S}^*_{G_2}=\mathcal{S}_{G_2}\mathcal{G}_{22}^{-3/2}=\Gamma|\Gamma|^{-1}\times[\langle\omega_2^{*2}\rangle(3\langle\omega_2^{*4}\rangle-2\langle\omega_2^{*2}\rangle^2)-\langle\omega_2^{*6}\rangle]\times(\langle\omega_2^{*4}\rangle-\langle\omega_2^{*2}\rangle^2)^{-3/2}.$$

When replacing the moments of vorticity by their numerical values, one obtains

$$\mathcal{S}^*_{G_2}\simeq 0.64\Gamma|\Gamma|^{-1}. \tag{32}$$

Therefore, in the case of zero initial variance of G_2, vorticity produces a skewness $\mathcal{S}^*_{G_2}$ of order unity, of the same sign as the forcing. In the general case $\mathcal{I}_\gamma\neq 1$, the analysis will be undertaken, as previously, in the small-time limit.

Note that, as in the case of second-order moments (Sec. II B), the regime of vanishing rotation is trivial; it is indeed easily demonstrated that $\mathcal{S}_{G_2}$ decays as $\exp(-3f_Dt)$ and that $\mathcal{S}^*_{G_2}$ remains constant. The regimes of vanishing and moderate diffusion are tackled in turn in the following.

1. $t^*\ll 1$ and $f_D^*\to 0$

In this regime, expansion of Eq. (B3) leads to

$$\mathcal{S}_{G_2}=\mathcal{S}_{G_2}(0)\left[1-3(1-\langle\omega_2^{*2}\rangle)\frac{t^{*2}}{8}+\left(\frac{7}{2}-\frac{13}{2}\langle\omega_2^{*2}\rangle+3\langle\omega_2^{*4}\rangle\right)\frac{t^{*4}}{64}\right]+3\Gamma^3(\mathcal{I}_\gamma-1)\times(\langle\omega_2^{*4}\rangle-\langle\omega_2^{*2}\rangle^2)\frac{t^{*4}}{32}+O(t^{*6}). \tag{33}$$

Examination of Eq. (33) indicates that vorticity makes the initial skewness decrease via the $O(t^{*2})$ term. Vorticity also amplifies the initial skewness but the corresponding term is of order $O(t^{*4})$ only. The last $O(t^{*4})$ term represents production of skewness by vorticity; as Γ^3 is multiplied by a positive factor, this term expresses that, in the case of zero initial skewness, vorticity implies a skewness the sign of which is the sign of forcing.

The normalized skewness is derived from Eqs. (16) and (33):

$$\mathcal{S}^*_{G_2}=\mathcal{S}_\gamma\left[1+3\frac{\mathcal{I}_\gamma-2}{\mathcal{I}_\gamma-1}(\langle\omega_2^{*4}\rangle-\langle\omega_2^{*2}\rangle^2)\frac{t^{*4}}{128}\right]+3\Gamma|\Gamma|^{-1}\times(\mathcal{I}_\gamma-1)^{-1/2}(\langle\omega_2^{*4}\rangle-\langle\omega_2^{*2}\rangle^2)\frac{t^{*4}}{32}+O(t^{*6}), \tag{34}$$

where $\mathcal{S}_\gamma$ is the initial normalized skewness.

As expressed by the last term of Eq. (34), vorticity produces a normalized skewness that has the same sign as forcing. It also appears that, depending on the value of $\mathcal{I}_\gamma$, vorticity either destroys (if $\mathcal{I}_\gamma<2$, i.e., in the case of a weak initial variance of G_2) or strengthens (if $\mathcal{I}_\gamma>2$, that is, the initial variance of G_2 is large) the initial skewness $\mathcal{S}_\gamma$.

2. $t^*\ll 1$ and $O(f_D^*)=O(1)$

In this case,

$$\mathcal{S}_{G_2}=\mathcal{S}_{G_2}(0)\left\{1-3f_D^* t^*+3\left[3f_D^{*2}-\frac{1}{4}(1-\langle\omega_2^{*2}\rangle)\right]\frac{t^{*2}}{2}-9f_D^*\left[f_D^{*2}-\frac{1}{4}(1-\langle\omega_2^{*2}\rangle)\right]\frac{t^{*3}}{2}+O(t^{*4})\right\}+3\Gamma^3(\mathcal{I}_\gamma-1)(\langle\omega_2^{*4}\rangle-\langle\omega_2^{*2}\rangle^2)\frac{t^{*4}}{32}+O(t^{*5}). \tag{35}$$

Again, vorticity produces a skewness displaying the sign of forcing [last $O(t^{*4})$ term]. In the case of a nonzero initial skewness, however, the prevailing mechanism is the decrease implied by diffusion and represented by a $O(t^*)$ term.

The normalized skewness is found to be

$$\mathcal{S}_{G_2}^*=\mathcal{S}_\gamma\left[1-\frac{35}{2}f_D^{*3}t^{*3}+O(t^{*4})\right]+3\Gamma|\Gamma|^{-1}\times(\mathcal{I}_\gamma-1)^{-1/2}(\langle\omega_2^{*4}\rangle-\langle\omega_2^{*2}\rangle^2)\frac{t^{*4}}{32}+O(t^{*5}). \tag{36}$$

The destruction of the initial skewness by diffusion is again the leading mechanism although it occurs at third order only. $\mathcal{S}_{G_2}^*$ is promoted by vorticity through a $O(t^{*4})$ term.

IV. DISCUSSION AND CONCLUDING REMARKS

The above approach is just aimed at revealing the influence of vorticity on second- and third-order moments of the gradient of a passive scalar in a simplified situation; the latter can be considered to mimic forcing of a scalar in an isotropic medium. The main assumptions (making the problem analytically tractable) consisted in restricting the analysis to dominating rotation and in representing diffusion effects by a linear model already used in other studies [26]. Statistics presented in Sec. III can be viewed as being conditioned on rotation. The results were furthermore discussed in the small-time limit and can be summarized as follows.

Regarding second-order moments of scalar gradient components:

In the framework of the model used, diffusion alone has no effect in the destruction of an initial anisotropy at the level of second-order moments.

The return to isotropy is governed by vorticity [Eqs. (19), (22), (27), and (29)]. This is most likely explained by the fact that, as shown in a preliminary qualitative analysis, vorticity both promotes second-order moments of scalar derivatives taken in directions normal to forcing ($\mathcal{G}_{11}$ and $\mathcal{G}_{33}$, here) and ensures redistribution of gradient fluctuations energy toward the second-order moment corresponding to the direction of forcing (i.e., from $\mathcal{G}_{11}$ and $\mathcal{G}_{33}$ toward $\mathcal{G}_{22}$). Note that in the presence of rotation, diffusion contributes to the destruction of anisotropy but does not prevail in this process [Eq. (29)];

Regarding third-order moments, the analysis is less straightforward:

Vorticity, on the one hand, always produces a skewness of the scalar gradient component corresponding to the direction of forcing (G_2); this is represented by a weak term, of order $O(t^{*4})$ [Eqs. (33)–(36)]. On the other hand, in the regime of vanishing diffusion ($f_D^*\to 0$), vorticity also implies destruction of a possible initial skewness $\mathcal{S}_{G_2}(0)$ [Eqs. (33)]. In this regime, however, depending on the intensity of the initial fluctuations, vorticity either destroys or reinforces the initial normalized skewness of G_2 [Eq. (34)];

In the regime of moderate diffusion $[O(f_D^*)=O(1)]$, both decays of skewness and its normalized value are ensured by diffusion, the corresponding terms being of orders $O(t^*)$ and $O(t^{*3})$, respectively [Eqs. (35) and (36)]. It is worth mentioning, however, that without rotation, diffusion would not make the initial normalized skewness decrease, at least in the framework of the present model.

The present simplified analysis does not allow one to draw definitive conclusions regarding passive scalar gradient statistics in real turbulence. Some qualitative trends can, however, be put forward. In particular, it can be suggested that, when anisotropy of the scalar field results from the existence of front structures oriented normally to a large-scale forcing, mechanisms implying destruction of this anisotropy exist (at least at the level of second- and third-order moments of scalar derivatives) and are likely to depend on the structures thickness (say, ℓ). Indeed, when the latter is of the order of the Kolmogorov's length scale η (the smallest thickness of the front structures if unity Schmidt number is assumed) then vorticity at scales larger than η has a negligible influence with respect to diffusion at the front scale and cannot oppose anisotropy. This can be thought as a regime in which diffusion tends to be the prevailing mechanism without, for all that, playing on anisotropy since, as mentioned previously, diffusion acting alone does not destroy anisotropy at the level of second-order moments nor does it make the normalized skewness decrease; in other words, one could say that the front structures are thickened by diffusion more rapidly than they rotate. On the other hand, vorticity at scales of order η is as important as diffusion and ensures the return to isotropy of second-order moments; simultaneously, in the presence of such a vorticity, diffusion destroys the skewness in a regime that would correspond to ''moderate diffusion'' in the present study $[O(f_D^*)=O(1)]$.

If the thickness ℓ of the front structures is appreciably larger than η (say, of the order of the Taylor's microscale) then, vorticity at scales smaller than ℓ has obviously no effect on the structures because it cannot make them rotate. At scales of order ℓ, the influence of diffusion with respect to vorticity scales as Re_ℓ^{-1} (where Re_ℓ is the Reynolds number at scale ℓ defined as $\mathrm{Re}_\ell\sim\omega_\ell\ell^2/\nu$). This implies that at large Reynolds number, the possible destruction of anisotropy at the level of front structures the thickness of which is larger than η is governed by vorticity (at the scale of the front thickness) in a way that would correspond to the regime of ''vanishing diffusion'' in the present study ($f_D^*\to 0$).

Of course, part of the vorticity at scales larger than ℓ should also contribute to the process.

When considering arbitrary values of the Schmidt number, the latter discussion also suggests that the larger the Schmidt number, the larger the action of vorticity on the front structures (and thereby on anisotropy) should be.

Finally, the above results lend support to the idea that both vorticity and diffusion may play a significant role in counteracting the strain-generated anisotropy of a scalar field when the latter is forced by a large-scale gradient in isotropic turbulence. In passing, anisotropy at the level of second-order moments and hence, of dissipation, is likely to be destroyed essentially by vorticity. The latter conclusions would require specific experiments or simulations to be confirmed.

APPENDIX A: STATISTICS OF VORTICITY ORIENTATION

The vorticity vector $\boldsymbol{\omega}$ is defined in the coordinate system determined by the orthogonal vector base $(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3)$ with $\mathbf{x}_3=\mathbf{x}_1\times\mathbf{x}_2$. If θ is the angle between $\boldsymbol{\omega}$ and its projection $\boldsymbol{\omega}_P$ on the $(\mathbf{x}_1,\ \mathbf{x}_3)$ plane and ϕ is the angle between $\mathbf{x}_1$ and $\boldsymbol{\omega}_P$, then

$$\omega_1^*=(1-s^2)^{1/2}\cos\phi, \tag{A1}$$

$$\omega_2^*=s, \tag{A2}$$

$$\omega_3^*=(1-s^2)^{1/2}\sin\phi, \tag{A3}$$

where $\omega_i^*=\omega_i/\omega$ and $s=\sin\theta$.

The components ω_i^* define a vector with a random, isotropic orientation if ϕ and s are random variables with uniform distributions over $[0,2\pi]$ and $[-1,1]$, respectively. The corresponding probability density functions are $P_\phi(\phi')=1/2\pi$ and $P_s(s')=1/2$ with ϕ' and s' being the sample variables. Any moment of components ω_i^* can thus be computed using Eqs. (A1)–(A3) and the probability distribution functions P_ϕ and P_s:

$$\langle\omega_1^{*m}\omega_2^{*n}\omega_3^{*p}\rangle=\frac{1}{4\pi}\int_{-1}^{1}(1-s'^2)^{(m+p)/2}s'^n ds' \times\int_0^{2\pi}\cos^m\phi'\sin^p\phi' d\phi'. \tag{A4}$$

It is found that (1) any moment with at least one odd exponent among m, n, and p is zero (which follows from isotropy); (2) $\langle\omega_i^{*2n}\rangle=1/(2n+1)$; (3) $\langle\omega_i^{*2}\omega_j^{*2}\rangle=1/15$ $(i\neq j)$; (4) etc....

APPENDIX B: EXACT EXPRESSIONS FOR THE SECOND- AND THIRD-ORDER MOMENTS

1. Second-order moments of G_1 and G_2

$$\begin{aligned}\mathcal{G}_{11}=&\frac{\Gamma^2}{(1+4f_D^{*2})^2}[\langle\omega_1^{*2}\omega_2^{*2}\rangle[1-\exp(-f_D^*t^*)\\&\times\{\cos(t^*/2)+2f_D^*\sin(t^*/2)\}]^2+\langle\omega_3^{*2}\rangle\\&\times[2f_D^*+\exp(-f_D^*t^*)\{\sin(t^*/2)-2f_D^*\cos(t^*/2)\}]^2]\\&+\Gamma^2(\mathcal{I}_\gamma-1)\exp(-2f_D^*t^*)\\&\times\Bigg[\langle\omega_1^{*2}\omega_2^{*2}\rangle\{1-\cos(t^*/2)\}^2\\&+\frac{\langle\omega_3^{*2}\rangle}{(1+4f_D^{*2})^2}\sin^2(t^*/2)\Bigg].\end{aligned} \tag{B1}$$

$$\begin{aligned}\mathcal{G}_{22}=&\frac{\Gamma^2}{(1+4f_D^{*2})^2}(\langle\omega_2^{*4}\rangle-\langle\omega_2^{*2}\rangle^2)[1-\exp(-f_D^*t^*)\\&\times\{\cos(t^*/2)+2f_D^*\sin(t^*/2)\}]^2+\Gamma^2(\mathcal{I}_\gamma-1)\\&\times\exp(-2f_D^*t^*)[(1-2\langle\omega_2^{*2}\rangle+\langle\omega_2^{*4}\rangle)\cos^2(t^*/2)\\&+2(\langle\omega_2^{*2}\rangle-\langle\omega_2^{*4}\rangle)\cos(t^*/2)+\langle\omega_2^{*4}\rangle].\end{aligned} \tag{B2}$$

In the above equations, $\mathcal{I}_\gamma=\langle\gamma^2\rangle/\Gamma^2$ and thus, $\mathcal{I}_\gamma\geqslant1$; $\mathcal{I}_\gamma=1$ corresponds to zero initial variance of G_2. Note that the initial conditions imply that $\mathcal{G}_{11}(0)=0$ and $\mathcal{G}_{22}(0)=\Gamma^2(\mathcal{I}_\gamma-1)$.

2. Third-order moment of G_2

$$\begin{aligned}\mathcal{S}_{G_2}=&\mathcal{S}_{G_2}(0)\exp(-3f_D^*t^*)[(1-3\langle\omega_2^{*2}\rangle+3\langle\omega_2^{*4}\rangle-\langle\omega_2^{*6}\rangle)\cos^3(t^*/2)+3(\langle\omega_2^{*2}\rangle-2\langle\omega_2^{*4}\rangle+\langle\omega_2^{*6}\rangle)\cos^2(t^*/2)\\&+3(\langle\omega_2^{*4}\rangle-\langle\omega_2^{*6}\rangle)\cos(t^*/2)+\langle\omega_2^{*6}\rangle]+\frac{\Gamma^3}{(1+4f_D^{*2})^3}[\langle\omega_2^{*2}\rangle(3\langle\omega_2^{*4}\rangle-2\langle\omega_2^{*2}\rangle^2)-\langle\omega_2^{*6}\rangle]\\&\times[(\cos(t^*/2)+2f_D^*\sin(t^*/2))\exp(-f_D^*t^*)-1]^3+\frac{3\Gamma^3}{1+4f_D^{*2}}(\mathcal{I}_\gamma-1)\{[\langle\omega_2^{*2}\rangle(\langle\omega_2^{*4}\rangle-2\langle\omega_2^{*2}\rangle)+2\langle\omega_2^{*4}\rangle-\langle\omega_2^{*6}\rangle]\\&\times\cos^2(t^*/2)-2[\langle\omega_2^{*2}\rangle(\langle\omega_2^{*4}\rangle-\langle\omega_2^{*2}\rangle)+\langle\omega_2^{*4}\rangle-\langle\omega_2^{*6}\rangle]\cos(t^*/2)+(\langle\omega_2^{*2}\rangle\langle\omega_2^{*4}\rangle-\langle\omega_2^{*6}\rangle)\}\\&\times[\{\cos(t^*/2)+2f_D^*\sin(t^*/2)\}\exp(-f_D^*t^*)-1]\exp(-2f_D^*t^*).\end{aligned} \tag{B3}$$

$\mathcal{S}_{G_2}(0)$ is the initial skewness of G_2. It is linked to the initial normalized skewness, $\mathcal{S}_\gamma = \mathcal{S}_{G_2}(0)\mathcal{G}_{22}^{-3/2}(0)$ by

$$\mathcal{S}_\gamma = \mathcal{S}_{G_2}(0)|\Gamma|^{-3}(\mathcal{I}_\gamma - 1)^{-3/2}.$$

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